PUMPING LEMMA FOR REGULAR SETS

Pumping lemma gives a method of pumping (generating) many input strings from a given string. As pumping lemma gives a necessary condition, it can be used to show that certain sets are not regular.

Pumping lemma:-

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton with n states. Let L be the regular set accepted by M. Let $w \in L$ and $|w| \geq m$. If $m \geq n$, then there exist x, y, z such that $w = xyz$, $y \neq \lambda$ and $xy^iz \in L$ for each $i \geq 0$.

PROOF:-

Let $w = a_1 a_2 \ldots a_m$, $m \geq n$

$$\delta(q_0, a_1 a_2 \ldots a_i) = q_i$$

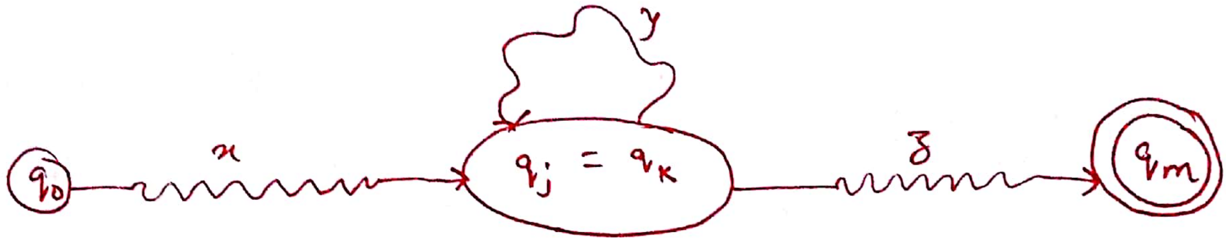
for $i = 1, 2, \ldots, m$

$$Q_1 = \{q_0, q_1, \ldots, q_m\}$$

That is, Q_1 is the sequence of states in the path with path value $w = a_1 a_2 \ldots a_m$.

As there are only n distinct states, at least two states in Q_1 must coincide. Among various pairs of repeated states, we take the first pair. Let us take them as q_j and q_k ($q_j = q_k$). Then j and k satisfy the condition $0 \leq j < k \leq n$.

The string w can be decomposed into three substrings $a_1 a_2 \dots a_j$, $a_{j+1} \dots a_k$ and $a_{k+1} \dots a_m$. let x, y, z denote these strings $a_1 a_2 \dots a_j$, $a_{j+1} \dots a_k$, $a_{k+1} \dots a_m$, respectively. As $k \leq n$, $|xy| \leq n$ and $w = xyz$. The path with path value w in the transition diagram of M is shown below:-



[Fig:- String accepted by M.]

The automaton M starts from the initial state q_0. On applying the string x, it reaches $q_j (= q_k)$. On applying the string y, it comes back to $q_j (= q_k)$. So after application of y^i for each $i \geq 0$, the automaton is in the same state q_j. On applying z it reaches q_m, a final state. Hence $xy^iz \in L$. As every state in Q_1 is obtained by applying an input symbol, $y \neq \lambda$.